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THE COMPUTATION OF SATURATION VAPOR PRESSURE

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The physical rationale of the concept of saturation vapor pressure is briefly discussed. Several procedures for the computation of saturation vapor pressure and its derivatives with respect to temperature are examined and evaluated with reference to accuracy relative to currently accepted standards and with reference to computational speed. It is demonstrated that it is possible to devise a set of 6th order polynomial

20. (continued)

approximations with better than satisfactory accuracy and,
further, that these formulas require considerably less computa-
tion time than other currently used procedures. Polynomials
are derived for various temperature ranges for both ice and
water references.

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DEFINITION OF SYMBOLS

p = Total pressure (mb)

E'_w = Saturation vapor pressure of moist air*

E_w = Saturation vapor pressure (over water)

E_I = Saturation vapor pressure (over ice)

E_s = Saturation vapor pressure (reference unspecified)

E_G = Saturation vapor pressure - Goff Gratch formulae (reference unspecified)

E_T = Saturation vapor pressure - Tetëns formulae (reference unspecified)

E_{TL} = Saturation vapor pressure - Table look-up procedure (reference unspecified)

E_L = Saturation vapor pressure - Polynomial formulae (reference unspecified)

D_G = Derivative of saturation vapor pressure with respect to temperature - Goff-Gratch formulae (reference unspecified)

D_T = Derivative of saturation vapor pressure with respect to temperature - Tetëns' formulae (reference unspecified)

D_L = Derivative of saturation vapor pressure with respect to temperature - polynomial formulae (reference unspecified)

ε = Ratio of molecular weights (.62197)

r_w = Mixing ratio of water vapor (water reference)

t = Temperature in degrees centigrade (Celsius)

T = Temperature in degrees absolute (Kelvin)

T_o = Ice point temperature (273.16°K) at standard atmosphere

T_s = Steam point temperature (373.16°K) at standard atmosphere

E_{ws} = Saturation vapor pressure of pure liquid water at steam point temperature (1 standard atmosphere = 1013.246 mb)

E_{IO} = Saturation vapor pressure of pure water ice at the ice point temperature (0.0060273 standard atmospheres = 6.1077 mb)

f_w = Correction factor for the departure of the mixture of air and water vapor from the ideal gas laws.

*All other symbols refer to pure phase

I. INTRODUCTION

In the many numerical models of atmospheric phenomena which take into consideration the thermodynamics of moist air, it is routinely necessary to calculate values of saturation vapor pressure one or more times at each grid point of a two or three dimensional computational domain at every time step. An examination of computation time for typical models would reveal that an inordinate amount of time is spent in the computation of vapor pressure. An algorithm for computing this parameter with accuracy, and which requires significantly less computation time than currently used procedures, is presented below.

2. SATURATION VAPOR PRESSURE - BACKGROUND

As Murray (1967)[1] pointed out, when changes in phase of water substance are to be considered, "... the saturation vapor pressure should be defined as the equilibrium vapor pressure ..." This quantity is a function not only of ambient temperature, but also of the probable impurities in water or ice substance, and the size and form of droplets and crystals. In practical situations, only the ambient temperatures can be determined with any degree of reality. This precludes the use of equilibrium vapor pressure in thermodynamic calculations. For most applications (particularly those not involving explicit microphysical calculation), it is satisfactory to employ saturation vapor pressure (with reference to plane surfaces of water and ice). An additional complication arises from the difference

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[1]Murray, F. W., 1967: On the computation of saturation vapor pressure. J. Appl. Meteor., Vol. 6, pp. 203-204.

between saturation vapor pressure of moist air and the saturation vapor pressure of water vapor in the pure phase. Saturation vapor pressure of the pure phase is a function of temperature only. Saturation vapor pressure of moist air is given by

$$E_W' = \left[\frac{r_W}{\varepsilon + r_W}\right] p \qquad \text{(List, 1958)}[2]$$

This can be contrasted with a similar formulation for the pure phase

$$E_W = f_W \left[\frac{r_W}{\varepsilon + r_W}\right] p = f_W E_W' \qquad \text{(List, 1958)}$$

where f_W is the ratio of the saturation vapor pressure for the pure phase to that for moist air. This difference arises from three related effects which have been noted by List (1958). This difference amounts to, at most, 0.5% in the ranges of meteorological pressure and temperature.

3. THE STANDARD - GOFF-GRATCH FORMULAS

The Goff-Gratch formulations are the internationally accepted procedures for computing saturation vapor pressure (in the pure phase) over plane surfaces of water and ice (12th Conference of Directors of IMO, Wash , D.C., 1947). These formulations are as follows:

$$\log_{10} E_W = -7.90298(T_s/T - 1) + 5.02808 \log_{10}(T_s/T)$$

$$-1.3816 \times 10^{-7}(10^{11.344(1 - T/T_s)} - 1) \qquad \text{for water} \qquad (3.1)$$

$$+8.1328 \times 10^{-3}(10^{-3.49149(T_s/T - 1)} - 1) + \log_{10} E_{ws},$$

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[2]List, R.J., 1958, Smithsonian Meteorological Tables (6th revision). Washington, D.C., The Smithsonian Institution, pp. 527.

$$\log_{10} E_I = -9.09718(T_0/T - 1) - 3.56654 \log_{10}(T_0/T)$$

$$+ \; 0.876793(1 - T/T_0) + \log_{10} E_{IO}, \quad \text{for ice} \qquad (3.2)$$

where:

E_W = saturation vapor pressure over a plane surface of pure ordinary liquid water (mb),

E_I = saturation vapor pressure of a plane surface of pure ordinary water ice (mb),

T = absolute (thermodynamic) temperature (°K),

T_s = steam-point temperature (373.16°K),

E_{WS} = saturation pressure of pure ordinary liquid water at steam-point temperature (1 standard atmosphere = 1013.246 mb),

E_{IO} = saturation pressure of pure ordinary water ice at ice-point temperature (0.0060273 standard atmosphere = 6.1071 mb). (Goff and Gratch, 1946)[3]

The forms of the functions above are quite unwieldy for computational purpose. Murray (1967) transformed them through simple operations into the following.

$$E_W = 7.95357242 \times 10^{10} \qquad (3.3)$$

$$X \exp \left\{ -18.1972839 \left(\frac{T_s}{T} \right) + 5.02808 \ln \left(\frac{T_s}{T} \right) \right.$$

$$- \; 70242.1852 \exp \left[\frac{-26.1205253}{(T_s/T)} \right]$$

$$\left. + \; 58.0691913 \exp \left[-8.03945282 \left(\frac{T_s}{T} \right) \right] \right\}, \quad (3)$$

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[3]Goff, J.A. and S. Gratch, 1946: Low-pressure properties of water from -110 to 212F. Trans. Amer. Soc. Heat. Vent. Eng., 52, 95-721.

and

$$E_I = 5.75185606 \times 10^{10} \exp \left\{ -20.947031 \left(\frac{T_o}{T} \right) \right.$$

$$\left. - 3.56654 \ln \left(\frac{T_o}{T} \right) - \frac{2.01889049}{(T_o/T)} \right\} \text{ (4)}$$

(3.4)

Goff and Gratch (1946) claimed a 2×10^{-2} percent uncertainty for the water reference formulation, Eq. (3.1), (above 0°C) and 3×10^{-2} percent for the ice reference formulation, Eq. (3.2). The uncertainty value for water reference does not apply to values below 0°C where no experimental data were available. Values in the range of 0 to -50°C were derived by direct extrapolation.

Murray's reformulations, Eqs. (3.3) and (3.4), of the Goff-Gratch formulas differed from the original Goff-Gratch Eqs. (3.1) and (3.2) by a maximum of 6×10^{-5} percent (at -25°C) and 3×10^{-5} percent (at -20°C) for water and ice references respectively.

4. TETËNS' FORMULAS

Murray's transformations of the original Goff-Gratch equations are still rather unwieldy from the standpoint of ease of computation. In this respect, the transforms, Eqs. (3.3) and (3.4), gain little over the original. A simpler formulation for determining these values is highly desirable. The formulation most used in the field of meteorology has been and is that of Magnus. Tetëns (1930)[4] gave this as

$$\log_{10} E_s = \frac{tu}{t+v} + w \tag{4.1}$$

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[4]Tetëns, O., 1930: Über einige meteorologische Begriffe. Z. Geophys., 6, 297-309.

where t is the temperature (°C), w = 0.7858 for vapor pressure in mb, and

$$u = 9.5 \quad \text{for ice; and} \quad u = 7.5 \quad \text{over water.}$$
$$v = 265.5 \qquad\qquad\qquad v = 237.3$$

A later statement of this formula can be found in Haurwitz (1945)[5] and is given as

$$E_s = 6.1078 \times 10^{\left(\frac{ut}{t+v}\right)} \tag{4.2}$$

where u, v and t have the values and meanings given above.

Murray (1967), for the purpose of achieving greater ease and speed of computation, reformulated Eq. (4.2) to

$$E_s = 6.1078 \, \exp \frac{a(T-273.16)}{(T-b)} \tag{4.3}$$

where T is temperature (°K), and

$$a = 21.8745584 \quad \text{for ice;} \quad a = 17.2693882 \quad \text{for water.}$$
$$b = 7.66 \qquad\qquad\qquad b = 35.86$$

Murray (1967) showed that the maximum difference between the Goff-Gratch and Tetëns formulation, for both ice and water, was well within the degree of uncertainty demonstrated by Goff and Gratch (1946). The amount of error (or difference) arising from the use of Tetëns formulation, Eq. (4.3), can be seen in Table 1 for water and Table 2 for ice. The maximum error (difference) for water is 4.4 percent at -50°C, and, for ice is 3.0 percent at -50°C.

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[5]Haurwitz, B., 1945, <u>Dynamic Meteorology</u>, New York, N. Y., McGraw-Hill, pp.

5. TABLE LOOK-UP PROCEDURE

In addition to the procedures discussed in the preceding sections, there is another method for computing saturation vapor pressure which is quite popular. This is the method of "table look-up." It is particularly attractive when considerable computer memory is available to a programmer. The "table look-up" procedure requires the storage of tabular values of saturation vapor pressure over a desired range of temperature. This stored table becomes a permanent and integral part of the program.

For a given temperature, limits of saturation vapor pressure are chosen from the table. This is accomplished by determining the algebraically largest tabular value of temperature less than the temperature in question. The saturation vapor pressures for this and the next higher tabular entries are chosen for limits (e.g., in a table of vapor pressure values for each whole degree temperature, for a temperature of 6.55°C, the limits will be the vapor pressure values for 6°C and 7°C). The required value of vapor pressure is then determined by linear (or higher order) interpolation within these limits. Linear interpolation is normally sufficient because values of vapor pressure between those for integer values of temperature are closely approximated by a straight line.

The accuracy attained by this procedure is very acceptable (see Table 1) with the largest error occurring at the middle of a tabular interval (i.e., at (T + 0.5)°C for a table of values at 1°C tabular intervals). The "table look-up" method is more accurate than the Tetëns' formulations for temperatures less than -5°C and usually less accurate for those above -5°C (using Goff-Gratch as the standard). It is slightly more accurate than the polynomial procedures (see section 6) from -50 to -25°C but less accurate above -25°C.

The major disadvantages of this procedure are the requirement for considerable memory storage and the necessity to compute interpolation limits which is time consuming (see Table 3).

6. A POLYNOMIAL APPROXIMATION

Even though the form of Tetëns' formula given by Eq.(4.3) and the procedure of table look-up are less complicated than the standard, they still leave something to be desired with respect to speed of computation. This is so because of the presence of the exponential function in Tetëns formulation which is quite time consuming even in those computers having a hard wired exponential algorithm (such as the CDC-6500). The table look-up procedure also consumes considerable time by requiring the determination of interpolation limits. Further, the procedure requires considerable memory storage. Clearly an even speedier procedure for the determination of saturation vapor pressure is highly desirable.

An examination of the curves for saturation vapor pressure indicated, because of the smooth variability with changing temperature, that they might be amenable to approximation by some order of polynomial without too much loss of accuracy. A least squares fitting procedure was applied to the data for the temperature range from -50°C to +50°C for the water reference and -50°C to 0°C for the ice reference. Polynomials of order one through five gave a very poor fit with maximum percentage errors of

18212	for order	1
9294	"	2
2854	"	3
530	"	4
48	"	5 .

All of these percentage errors occurred at -50°C (for the water reference). The maximum percentage errors for the ice reference for polynomials of order one through five are

1540	for order	1
601	"	2
149	"	3
26	"	4
2.6	"	5 .

The sixth order polynomials for both the ice and liquid water reference gave errors of less than one percent for the entire meteorological range of interest. The polynomial formulation for saturation vapor pressure is

$$E_s = a_0 + t \, (a_1 + t(a_2 + t(a_3 + t(a_4 + t(a_5 + a_6 t))))), \qquad (6.1)$$

where t is temperature in degrees centigrade* and the constants have the following values

for water	for ice
$a_0 = 6.107799961$	$a_0 = 6.109177956$
$a_1 = 4.436518521 \times 10^{-1}$	$a_1 = 5.034698970 \times 10^{-1}$
$a_2 = 1.428945805 \times 10^{-2}$	$a_2 = 1.886013408 \times 10^{-2}$
$a_3 = 2.650648471 \times 10^{-4}$	$a_3 = 4.176223716 \times 10^{-4}$
$a_4 = 3.031240396 \times 10^{-6}$	$a_4 = 5.824720280 \times 10^{-6}$
$a_5 = 2.034080948 \times 10^{-8}$	$a_5 = 4.838803174 \times 10^{-8}$
$a_6 = 6.136820929 \times 10^{-11}$	$a_6 = 1.838826904 \times 10^{-10}$

(Range of validity: -50°C to +50°C for water, -5C C + 0°C for ice)

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*The coefficients can be readily re-evaluated for use with temperatures in degrees Kelvin.

7. ACCURACY RELATIVE TO THE ACCEPTED STANDARD

Table 1 gives values of saturation vapor pressure (over water) as calculated by the Goff-Gratch formulation (E_G), Eq. (3.1); by Tetëns' formula (E_T), Eq. (4.3); by table look-up (E_{TL}); and by the polynomial (E_L), Eq. (5.1). Also shown in Table 1 are values of percentage departure (error) of the Tetëns formula and polynomial results from the Goff-Gratch standard. These percentages are indicated by the values in parentheses. Table 2 gives analogous information for saturation vapor pressure with respect to a plane ice surface. A quick examination of these tables indicates that, with only one exception (0°C, for ice, Table 2), the percentage departure due to the polynomial procedure is everywhere many times less than that due to the Tetëns formulation. Therefore, as Murray (1967) has shown, saturation vapor pressure values determined by Tetëns formula, Eq. (4.3), depart from the standard by amounts less than the degree of uncertainty embodied in the standard. The polynomial values which have smaller departures must be even further within the zone of uncertainty.

Having shown that the polynomial yields values of saturation vapor pressure which are <u>at least as accurate</u> as the Tetëns formulations, it is next necessary to inquire into the relative speeds of computation of the methods discussed above.

Each of the procedures was used to compute a set of 10,000 saturation vapor pressures. Evaluations were made on two computer systems -- the CDC 3100 and the CDC 7600. The results of these evaluations are shown in Table 3.

From Table 3, it can be seen that the polynomial formulation is approximately 2.5 times faster than the best Tetëns formulation. It would seem, then, that the demonstrated accuracy and speed would justify the use of the polynomial for the determination of saturation vapor pressure in numerical models.

Table 1. Values of saturation vapor pressure (in mb) (over water). (Values in () are % error.)

T(°C)	E_G	E_T	*E_{TL}	E_L
-50	0.06356	0.06078(-4.4)	.0674(0.15)	0.06337(-0.29)
-45	0.11114	0.10735(-3.4)	.1175(0.085)	0.11170(+0.54)
-40	0.18914	0.18423(-2.6)	.1994(0.10)	0.18915(+0.03)
-35	0.31387	0.30783(-1.9)	.3301(0.12)	0.31314(-0.24)
-30	0.50880	0.50177(-1.38)	.5338(0.10)	0.50777(-0.20)
-25	0.80697	0.79928(-0.95)	.8448(0.09)	0.80620(-0.10)
-20	1.25401	1.24622(-0.62)	1.3102(0.08)	1.25386(-0.01)
-15	1.91178	1.90463(-0.37)	1.9936(0.08)	1.91226(+0.02)
-10	2.86270	2.85709(-0.196)	2.9799(0.07)	2.86350(+0.028)
-5	4.21485	4.21168(-0.075)	4.3800(0.07)	4.21548(+0.016)
0	6.1078	6.1078(0.00)	6.3370(0.06)	6.1078(0.00)
+5	8.71922	8.72272(+0.04)	9.0328(0.05)	8.71839(-0.009)
+10	12.2723	12.2789(+0.054)	12.696(0.04)	12.2707(-0.010)
+15	17.0438	17.0523(+0.050)	17.608(0.05)	17.0419(-0.012)
+20	23.3730	23.3810(+0.034)	24.117(0.04)	23.3712(-0.008)
+25	31.6709	31.6749(+0.013)	32.640(0.04)	31.6693(-0.005)
+30	42.4304	42.4264(-0.009)	43.678(0.04)	42.4289(-0.002)
+35	56.2366	56.2206(-0.028)	57.829(0.03)	56.2348(-0.002)
+40	73.7775	73.7473(-0.041)	75.790(0.03)	73.7754(-0.002)
+45	95.8548	95.8125(-0.044)		95.8553(+0.0003)
+50	123.400	123.351(-0.036)		123.408(+0.006)

*Temperature for this column are offset upwards by 1/2 degree, i.e., the 0°C entry is really the value for +0.5°C.

Table 2. Values of saturation vapor pressure (mb) (over ice). (Values in ( ) are % error.)

T(°C)	E_G	E_T	E_L
-50°	0.03935	0.03817(-3.0)	0.03963(+0.713)
-45	0.07198	0.07032(-2.3)	0.07192(-0.076)
-40	0.12832	0.12611(-1.7)	0.12830(-0.002)
-35	0.22329	0.22048(-1.26)	0.22315(-0.066)
-30	0.37980	0.37644(-0.89)	0.37964(-0.041)
-25	0.63233	0.62857(-0.59)	0.63230(-0.000)
-20	1.03173	1.02790(-0.37)	1.03174(-0.026)
-15	1.65167	1.64824(-0.21)	1.65141(-0.035)
-10	2.59702	2.59456(-0.095)	2.59646(-0.021)
-5	4.01465	4.01358(-0.024)	4.01462(-0.009)
0	6.10710	6.1078(+0.011)	6.10918(+0.035)

Table 3. Comparison of computation times for SVP procedures (over water) (units = sec/computation).

Procedure		3100	7600
E_G	(3.3)	16.316×10^{-4}	20.331×10^{-6}
E_{T_1}	(4.2)	9.849×10^{-4}	11.400×10^{-6}
E_{T_2}	(4.3)	4.583×10^{-4}	5.76×10^{-6}
E_{TL}		4.975×10^{-4}	----------
E_L	(6.1)	1.926×10^{-4}	2.400×10^{-6}

8. COMPUTATION OF THE DERIVATIVE WITH RESPECT TO TEMPERATURE

Many thermodynamic computations necessary for atmospheric simulation require determination of values of the derivative of saturation vapor pressure with respect to temperature. Differentiation of the Goff-Gratch equations as reformulated by Murray (see Eqs. 3.3 and 3.4) yields

$$\frac{dE_W}{dT} = -\left[5.02808 - \left(18.1973 + 446.844 \exp\left\{-8.03945\left(\frac{T_s}{T}\right)\right\}\right)\left(\frac{T_s}{T}\right) - \frac{1834762}{(T_s/T)} \exp\left\{\frac{-26.1205}{(T_s/T)}\right\}\right]\frac{E_W}{T} \qquad (8.1)$$

for liquid

$$\frac{dE_I}{dT} = \left[20.947\left(\frac{T_o}{T}\right) + 3.56654 - \frac{2.0189}{(T_o/T)}\right]\frac{E_I}{T} \qquad \text{for ice} \quad (8.2)$$

(Murray & Hollinden, 1966)[6]

It can be seen that these expressions are much more complicated than even the original Goff-Gratch equations. Besides the complication of form, it is also required to calculate the saturation vapor pressure itself, if it is not already known -- which is not likely.

Logarithmic differentiation of Tetëns' formula, Eq. (4.3), gives

$$\qquad (8.3)$$

$$\frac{dE_s}{dT} = \frac{A'E_s}{(T-B)^2} \quad \text{where } A' = 5807.71 \text{ over ice and } 4098.03$$

over water

$$B = 35.86 \text{ over water and } 7.66 \text{ over ice.}$$

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[6]Murray, F.W. and A.B. Hollinden, 1966: The evaluation of cumulus clouds: A numerical simulation and its comparison against observations. Douglas Aircraft Co. Rep. #SM-49372.

This expression would lead to a very rapid calculation of the derivative if E_s is known. If it is not, then the calculation will be slightly more lengthy than Tetëns' calculation for saturation vapor pressure.

An attempt was made to fit a polynomial to values of the derivative over the range of temperatures of meteorological interest. The data used was obtained by evaluations of Eqs. (8.1) and (8.2). Polynomials of order 5, 6 and 7 all showed acceptable error patterns (i.e., errors less than those arising from the Tetëns formulation, Eq. (8.3)) for water. For ice reference, orders 5 and 6 were acceptable, but strangely enough higher orders were not. An evaluation of mean, maximum and root-mean-square errors indicated that the sixth order polynomial was again the optimum choice. The polynomials take the same form as Eq. (6.1) but with coefficients as shown below:

for water	for ice	(8.4)
$a_0 = 4.438099984 \times 10^{-1}$	$a_0 = 5.030305237 \times 10^{-1}$	
$a_1 = 2.857002636 \times 10^{-2}$	$a_1 = 3.773255020 \times 10^{-2}$	
$a_2 = 7.938054040 \times 10^{-4}$	$a_2 = 1.267995369 \times 10^{-3}$	
$a_3 = 1.215215065 \times 10^{-5}$	$a_3 = 2.477563108 \times 10^{-5}$	
$a_4 = 1.036561403 \times 10^{-7}$	$a_4 = 3.005693132 \times 10^{-7}$	
$a_5 = 3.532421810 \times 10^{-10}$	$a_5 = 2.158542548 \times 10^{-9}$	
$a_6 = -7.090244804 \times 10^{-13}$	$a_6 = 7.131097725 \times 10^{-12}$	

(range of validity: $-50°C$ to $+50°C$ for water;

$-50°C$ to $0°C$ for ice)

Table 4 shows the values of $\frac{dE_s}{dT}$ as computed using the Goff-Gratch (D_G) derivatives, Tetëns' derivative (D_T), and those determined from the polynomial (D_L) just discussed. Table 5 shows similar information for dE_s/dT for the ice reference case. As with the primary functions, the polynomial expressions, coefficient set (8.4), for the derivatives (with reference to both ice and water) show departures (errors) from the Goff-Gratch standard which are considerably less than those determined by the use of Tetëns' formulation, Eq. (8.3). Derivative computation times are comparable (as would be expected) to those for the primary expressions for saturation vapor pressure (see Table 3).

9. SUMMARY

It has been shown that it is possible to formulate a polynomial approximation for both saturation vapor pressure and its derivative with respect to temperature that is at least and 's generally much more accurate than currently used procedures (Tetëns' formula). Accuracy was measured in terms of departure from values derived from the Goff-Gratch formulas which are the internationally accepted standards. The polynomial errors are well within the degree of uncertainty connected with the Goff-Gratch procedures. The polynomial procedures have been demonstrated to consume significantly less computer time than methods currently in use. The employment of this procedure will result in significant savings in the consumption of computer resources and money. These same polynomials may be used to evaluate actual vapor pressure by using the dew point temperature in lieu of air temperature.

Table 4. Derivative of saturation vapor pressure over water (mb/degree).

$T(°C)$	D_G	D_T	D_L
-50°C	0.007286	0.007100(-2.6)	0.007188(-1.35)
-45°C	0.012113	0.011897(-1.8)	0.012234(+1.001)
-40°C	0.019624	0.019394(-1.17)	0.019644(+0.099)
-35°C	0.031042	0.030824(-0.70)	0.030940(-0.329)
-30°C	0.048021	0.047849(-0.36)	0.047887(-0.279)
-25°C	0.072756	0.072673(-0.11)	0.072678(-0.107)
-20°C	0.10811	0.10816(+0.045)	0.10812(+0.013)
-15°C	0.15773	0.15795(+0.136)	0.15782(+0.059)
-10°C	0.22622	0.22662(+0.176)	0.22634(+.052)
-5°C	0.31927	0.31983(+0.179)	0.31935(+.025)
0°C	0.44381	0.44449(+.154)	0.44381(0.00)
5°C	0.60817	0.60886(0.114)	0.60809(-0.013)
10°C	0.82225	0.82279(0.065)	0.82211(-0.016)
15°C	1.0976	1.0978(0.016)	1.0975(-0.010)
20°C	1.4477	1.4473(-0.028)	1.4476(-0.005)
24°C	1.8878	1.8867(-0.063)	1.8878(+0.002)
30°C	2.4354	2.4334(-0.083)	2.4355(+0.003)
35°C	3.1100	3.1072(-0.087)	3.1100(-0.0002)
40°C	3.9331	3.9303(-0.072)	3.9331(-.0009)
45°C	4.9287	4.9269(-0.036)	4.9286(-0.001)
50°C	6.1228	6.1241(+.022)	6.1230(+.003)

Table 5. Derivative of saturation vapor pressure over ice (mb/degree).

T(°C)	D_G	D_T	D_L
-50°C	0.004859	0.004773(-1.8)	0.004875(+0.32)
-45°C	0.008505	0.008400(-1.23)	0.008502(-0.033)
-40°C	0.014521	0.014403(-0.81)	0.014512(-0.059)
-35°C	0.024219	0.024101(-0.49)	0.024210(-0.036)
-30°C	0.039519	0.039420(-0.25)	0.039515(-0.010)
-25°C	0.063166	0.063115(-0.081)	0.063166(-0.000)
-20°C	0.099022	0.099050(+0.028)	0.099013(-0.009)
-15°C	0.15241	0.15255(+0.089)	0.152381(-0.018)
-10°C	0.23058	0.23083(+0.109)	0.23053(-0.023)
- 5°C	0.34317	0.34350(+0.097)	0.34315(-0.005)
0°C	0.50292	0.50322(+0.061)	0.50303(+0.022)

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APPENDIX

SATURATION VAPOR PRESSURE AT VERY LOW TEMPERATURES

The polynomial approximations discussed and evaluated in the foregoing were strictly valid only for given ranges of temperature (-50°C to +50°C for water reference and -50°C to 0°C for ice reference). Attempts to use these polynomials at lower temperatures resulted in drastic departures from the Goff-Gratch values. To overcome this difficulty, values of saturation vapor pressure (with respect to both ice and water) were generated from the Goff-Gratch formulas. These data were then used to derive a new set of polynomial approximations. The polynomials are again of the sixth order and take the same form as that in Eq. (6.1) with the coefficients shown below:

for water	for ice
$a_0 = 4.866786841$	$a_0 = 3.927659727$
$a_1 = 3.152625546 \times 10^{-1}$	$a_1 = 2.643578680 \times 10^{-1}$
$a_2 = 8.640188586 \times 10^{-3}$	$a_2 = 7.505070860 \times 10^{-3}$
$a_3 = 1.279669658 \times 10^{-4}$	$a_3 = 1.147668232 \times 10^{-4}$
$a_4 = 1.077955914 \times 10^{-6}$	$a_4 = 9.948650743 \times 10^{-7}$
$a_5 = 4.886796102 \times 10^{-9}$	$a_5 = 4.626362556 \times 10^{-9}$
$a_6 = 9.296950850 \times 10^{-12}$	$a_6 = 9.001382935 \times 10^{-12}$

(Range of validity: -100°C to -50°C)

Table A-1 shows the accuracy of the approximation with respect to water as compared to that of Tetëns using Goff-Gratch as a standard. Percentage errors are shown in parentheses. With the exception of -100°C, the polynomial values depart from the Goff-Gratch values by a considerably smaller percentage than

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those derived from Tetëns formulation. Table A-2, shows
similar information for saturation vapor pressure with respect
to ice reference.

Table A-1. Values of Saturation Vapor Pressure (mb)
(water reference) at very low temperatures.

t(°C)	E_G	E_T	E_L
-100	$.24 \times 10^{-4}$	$.21 \times 10^{-4}(-12.25)$	$.33 \times 10^{-4}(36.14)$
-95	$.70 \times 10^{-4}$	$.60 \times 10^{-4}(-14.13)$	$.68 \times 10^{-4}(-2.49)$
-90	$.187 \times 10^{-3}$	$.160 \times 10^{-3}(-14.41)$	$.192 \times 10^{-3}(+3.13)$
-85	$.462 \times 10^{-3}$	$.398 \times 10^{-3}(-13.77)$	$.458 \times 10^{-3}(-0.72)$
-80	$.107 \times 10^{-2}$	$.936 \times 10^{-3}(-12.60)$	$.106 \times 10^{-2}(-0.74)$
-75	$.235 \times 10^{-2}$	$.209 \times 10^{-2}(-11.17)$	$.235 \times 10^{-2}(+0.00)$
-70	$.492 \times 70^{-2}$	$.444 \times 10^{-2}(-9.65)$	$.492 \times 10^{-2}(+0.07)$
-65	$.985 \times 10^{-2}$	$.905 \times 10^{-2}(-8.16)$	$.984 \times 10^{-2}(-0.13)$
-60	.01898	.01769(-6.76)	.01894(-0.18)
-55	.03529	.03335(-5.49)	.03528(-0.03)
-50	0.06356	.06078(-4.37)	.06362(+0.11)

Table A-2. Values of Saturation Vapor Pressure (mb) (ice
reference) at very low temperatures.

t(°C)	E_G	E_T	E_L
-100	$.1403 \times 10^{-4}$	$.1114 \times 10^{-4}(-20.61)$	$.2313 \times 10^{-4}(+64.85)$
-95	$.3784 \times 10^{-4}$	$.3117 \times 10^{-4}(-17.62)$	$.3587 \times 10^{-4}(-5.20)$
-90	$.9672 \times 10^{-4}$	$.8224 \times 10^{-4}(-14.96)$	$.1047 \times 10^{-3}(+8.23)$
-85	$.235 \times 10^{-3}$	$.2056 \times 10^{-3}(-12.61)$	$.2321 \times 10^{-3}(-1.39)$
-80	$.5472 \times 10^{-3}$	$.4894 \times 10^{-3}(-10.55)$	$.5375 \times 10^{-3}(-1.78)$
-75	$.1220 \times 10^{-2}$	$.1113 \times 10^{-2}(-8.75)$	$.1222 \times 10^{-2}(+0.02)$
-70	$.2615 \times 10^{-2}$	$.2427 \times 10^{-2}(-7.19)$	$.2622 \times 10^{-2}(+0.29)$
-65	$.5406 \times 10^{-2}$	$.5090 \times 10^{-2}(-5.84)$	$.5396 \times 10^{-2}(-0.17)$
-60	$.1080 \times 10^{-1}$	$.1030 \times 10^{-1}(-4.69)$	$.1077 \times 10^{-1}(-0.32)$
-55	$.2092 \times 10^{-1}$	$.2015 \times 10^{-1}(-3.71)$	$.2091 \times 10^{-1}(-0.06)$
-50	$.3935 \times 10^{-1}$	$.3821 \times 10^{-1}(-2.89)$	$.3940 \times 10^{-1}(+0.15)$

The polynomial approximations for the derivative of saturation vapor pressure with respect to temperature for ice and water reference is also of sixth order and have the same form as Eq. (6.1).

for water

$$a_0 = 4.086240791 \times 10^{-1}$$

$$a_1 = 2.516118369 \times 10^{-2}$$

$$a_2 = 6.576862688 \times 10^{-4}$$

$$a_3 = 9.325531513 \times 10^{-6}$$

$$a_4 = 7.550718726 \times 10^{-8}$$

$$a_5 = 3.303373957 \times 10^{-10}$$

$$a_6 = 6.088242842 \times 10^{-13}$$

for ice

$$a_0 = 3.808584154 \times 10^{-1}$$

$$a_1 = 2.482536693 \times 10^{-2}$$

$$a_2 = 6.847888228 \times 10^{-4}$$

$$a_3 = 1.020750000 \times 10^{-5}$$

$$a_4 = 8.651642035 \times 10^{-8}$$

$$a_5 = 3.944818571 \times 10^{-10}$$

$$a_6 = 7.544801596 \times 10^{-13}$$

(Range of validity: -100°C to -50°C)

Tables A-3 and A-4 show the comparative accuracy of polynomial approximations and Tetëns' formula for the derivative of vapor pressure with respect to water and ice reference. It is interesting to compare the juncture (-50°C) of the tables in this appendix with the corresponding tables in the main body of the text (Table A-1 corresponds to Table 1; A-2 to 2; A-3 to 4; and, A-4 to 5). For vapor pressure with respect to water, the "normal" value (normal implies an ordinary meteorological temperature range as opposed to the very low temperature range) is .06337 mb (-.29) while the low range value is .06362 mb (+0.11). The absolute percentage difference between these values is 0.39% with the low range value being the more accurate with respect to the Goff-Gratch values. For vapor pressure with respect to ice, this percentage is 2.58%; for the derivative with reference to water, 1.45%; and lastly, for the derivative with reference to ice, 0.23%. In all cases,

Table A-3. Values of derivative of saturation vapor pressure (mb/deg) with respect to temperature (water reference) at very low temperatures.

$t(°C)$	D_G	D_T	D_L
-100	$.538 \times 10^{-5}$	$.458 \times 10^{-5}(-14.91)$	$.593 \times 10^{-5}(+10.28)$
-95	$.1432 \times 10^{-4}$	$.1216 \times 10^{-4}(-15.05)$	$.1421 \times 10^{-4}(-0.75)$
-90	$.3514 \times 10^{-4}$	$.3017 \times 10^{-4}(-14.13)$	$.3530 \times 10^{-4}(+0.47)$
-85	$.8055 \times 10^{-4}$	$.7035 \times 10^{-4}(-12.66)$	$.8029 \times 10^{-4}(-0.32)$
-80	$.1742 \times 10^{-3}$	$.1551 \times 10^{-3}(-10.95)$	$.1738 \times 10^{-3}(-0.22)$
-75	$.3581 \times 10^{-3}$	$.3251 \times 10^{-3}(-9.20)$	$.3581 \times 10^{-3}(+0.00)$
-70	$.7037 \times 10^{-3}$	$.6507 \times 10^{-3}(-7.53)$	$.7037 \times 10^{-3}(-0.00)$
-65	$.1329 \times 10^{-2}$	$.1249 \times 10^{-2}(-6.00)$	$.1327 \times 10^{-2}(-0.09)$
-60	$.2419 \times 10^{-2}$	$.2307 \times 10^{-2}(-4.65)$	$.2417 \times 10^{-2}(-0.10)$
-55	$.4262 \times 10^{-2}$	$.4112 \times 10^{-2}(-3.51)$	$.4262 \times 10^{-2}(-0.02)$
-50	$.7286 \times 10^{-2}$	$.7100 \times 10^{-2}(-2.55)$	$.7291 \times 10^{-2}(+0.07)$

Table A-4. Values of derivative of saturation vapor pressure (mb/deg) with respect to temperature (ice reference) at very low temperatures.

$t(°C)$	D_G	D_T	D_L
-100	$.286 \times 10^{-5}$	$.236 \times 10^{-5}(-17.50)$	$.358 \times 10^{-5}(+25.18)$
-95	$.730 \times 10^{-5}$	$.623 \times 10^{-5}(-14.70)$	$.715 \times 10^{-5}(-2.03)$
-90	$.1766 \times 10^{-4}$	$.1550 \times 10^{-4}(-12.23)$	$.1819 \times 10^{-4}(+2.95)$
-85	$.4076 \times 10^{-4}$	$.3665 \times 10^{-4}(-10.07)$	$.4048 \times 10^{-4}(-0.67)$
-80	$.8997 \times 10^{-4}$	$.8259 \times 10^{-4}(-8.20)$	$.8929 \times 10^{-4}(-0.76)$
-75	$.1907 \times 10^{-3}$	$.1781 \times 10^{-3}(-6.60)$	$.1907 \times 10^{-3}(+0.00)$
-70	$.3891 \times 10^{-3}$	$.3687 \times 10^{-3}(-5.24)$	$.3895 \times 10^{-3}(+0.09)$
-65	$.7665 \times 10^{-3}$	$.7352 \times 10^{-3}(-4.08)$	$.7655 \times 10^{-3}(-0.13)$
-60	$.1462 \times 10^{-2}$	$.1416 \times 10^{-2}(-3.12)$	$.1459 \times 10^{-2}(-0.19)$
-55	$.2703 \times 10^{-2}$	$.2640 \times 10^{-2}(-2.33)$	$.2702 \times 10^{-2}(-0.03)$
-50	$.4859 \times 10^{-2}$	$.4777 \times 10^{-2}(-1.68)$	$.4864 \times 10^{-2}(+0.11)$

the cold range value is the more accurate. The important point to make, however, is that the polynomials for both the "normal" and the very low ranges may be used together without suffering a severe or significant discontinuity in variation in the.curve of values (both zero and first order).

Finally, computation time for the various procedures are comparable tc those shown in Table 3 and maintain the same relativity.